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Studies on Heavy Particle Decay from $^{216\text{-}226}$Fr using Different Nuclear Potentials

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Abstract: *The heavy particle decays from various isotopes of $^{216\text{-}226}$Fr have been studied by evaluating the decay half lives using the Coulomb and proximity potential model (CPPM) and also using the nuclear potentials, Bass 1973, Bass 1977, Bass 1980 and BW 1991. Comparing the calculated values with the available experimental data, it is observed that the calculations using CPPM and Bass potential, the Bass 1973, are in excellent agreement with the experimental value. All other nuclear potentials; Bass 1977, Bass 1980 and BW 1991; showed one order difference in half lives with the experimental value. The existence of the well established magic number, N=126, is obtained from the plot of $\log_{10}(T_{1/2})$ against the neutron number of the daughter nuclei. The Geiger-Nuttall (G-N) plots of $\log_{10}T_{1/2}$ vs. $Q^{-1/2}$ and -lnP vs. $ZQ^{-1/2}$ are obtained with same slope and different intercepts for CPPM and other nuclear potentials. The Universal curve of $\log_{10}T_{1/2}$ vs. –ln P studied for various nuclear potentials are also obtained as linear.*

Keywords – *Heavy particle radioactivity, Geiger-Nuttall law, Universal curve, Proximity potential.*

I. Introduction

The possibility of emission of clusters heavier than alpha particle but lighter than typical binary fission fragments from an unstable nuclide, named as heavy particle radioactivity (HPR) or cluster radioactivity, was first described by Sandulescu et al. [1], in 1980. The experimental confirmation of HPR was first given by Rose and Jones [2] by the detection of ^{14}C from ^{223}Ra in a huge background of alpha particles. Later on, several clusters were observed experimentally from various parents in the trans-lead region with partial half-lives from 10^{11} up to 10^{30}s and branching ratios relative to alpha decay from 10^{-9} down to 10^{-19}. Till now, 24 cases of spontaneous emission of clusters ranging from ^{14}C to ^{34}Si [3] from various parent nuclei have been detected.

Heavy particle radioactivity can be described using two main formalisms, alpha-like approach [4] and fission like approach [5, 6]. In the former, the probability of cluster formation is determined by the overlap of the parent nucleus wave function with those of decay fragments resulting in a sudden formation of a cluster and tries to penetrate the Coulomb barrier. In the later, heavy particle decay is considered to be a single step process. It includes the pre-scission phase where the fragments are overlapping. Here, an exotic nucleus is considered to split up into two asymmetric fragments.

Santhosh et al., [7, 8] have calculated half-lives for experimentally observed cluster decay modes of several heavy nuclei in the trans-lead and trans-tin region by using Coulomb proximity potential model (CPPM) [9, 10]. The present work is an extension of our earlier work [11] which presents a comparative study of heavy particle decay using different nuclear potentials. We have undertaken heavy particle decay half life evaluations for the emission of ^{14}C cluster from the heavy nuclei $^{216\text{-}226}$Fr using CPPM and also using the nuclear potentials Bass 1973 [12], Bass 1977 [13], Bass 1980 [14] and BW 1991 [14].

A brief description of our model is presented in section 2. The details of the study carried out by us are provided in section 3, results and discussions, of the paper. In the last section, section 4, we summarized our main conclusions.

II. Model

2.1 Coulomb and proximity potential model (CPPM)

In the Coulomb and proximity potential model (CPPM), the potential energy barrier is taken as the sum of Coulomb potential, proximity potential and centrifugal potential for the touching configuration and for the separated fragments. For the pre-scission (overlap) region, simple power law interpolation as done by Shi and Swiatecki [6] is used. The inclusion of proximity potential reduces the height of the potential barrier, which closely agrees with the experimental result.

The interacting potential barrier for a parent nucleus exhibiting cluster decay is given by:

$$V = \frac{Z_1 Z_2 e^2}{r} + V_p(z) + \frac{\hbar^2 \ell(\ell+1)}{2\mu r^2} \quad \text{, for } z > 0 \tag{1}$$

Here Z_1 and Z_2 are the atomic numbers of the daughter and emitted cluster, 'z' is the distance between the near surfaces of the fragments, 'r' is the distance between fragment centers and is given as $r = z + C_1 + C_2$, where, C_1 and C_2 are the Süsmann central radii of fragments. The term ℓ represents the angular momentum, μ the reduced mass and V_P is the proximity potential. The proximity potential V_P is given by Blocki et al., [15, 16].

$$V_p(z) = 4\pi\gamma b\left[\frac{C_1 C_2}{(C_1 + C_2)}\right]\Phi\left(\frac{z}{b}\right) \tag{2}$$

With γ as the nuclear surface tension coefficient given as:

$$\gamma = 0.9517\,[1 - 1.7826\,(N - Z)^2 / A^2]\ \text{MeV/fm}^2 \tag{3}$$

Where N, Z and A represent neutron, proton and mass number of parent respectively, Φ represents the universal proximity potential [16].

Using one dimensional WKB approximation, the barrier penetrability P is given as:

$$\mathrm{P} = \exp\left\{-\frac{2}{\hbar}\int_a^b \sqrt{2\mu(V - Q)}\,dz\right\} \tag{4}$$

The turning points 'a' and 'b' are determined from the equation $V(a) = V(b) = Q$. The above integral can be evaluated numerically or analytically, and the half life time is given by:

$$T_{1/2} = \left(\frac{\ln 2}{\lambda}\right) = \left(\frac{\ln 2}{\nu P}\right) \tag{5}$$

Where, $\nu = \left(\frac{\omega}{2\pi}\right) = \left(\frac{2E_v}{h}\right)$ represent the number of assaults on the barrier per second and λ the decay constant and E_v is the empirical vibration energy.

1.2 Bass 1973

This model [12] is based on the assumption of liquid drop model. According to the model, the nuclear part of the interaction potential is given as:

$$V_N(r) = -\frac{d}{R_{12}} a_s A_1^{1/3} A_2^{1/3} \exp\left(-\frac{r - R_{12}}{d}\right) MeV \tag{6}$$

With $R_{12} = r_o(A_1^{1/3} + A_2^{1/3})$, d=1.35fm, a_s=17.0 MeV and r_0=1.07fm.

1.3 Bass 1977

In this model [13], based on the information from the experimental fusion cross sections by using the liquid drop model and using the general geometrical arguments, the nucleus-nucleus potential can be written as:

$$V_N(r) = -\frac{R_1 R_2}{R_1 + R_2}\Phi(r - R_1 - R_2) MeV \tag{7}$$

Here $\Phi(s = r - R_1 - R_2)$ is the universal function. The radius R_i written as:

$$R_i = 1.16\,A_i^{1/3} - 1.39\,A_i^{-1/3}\,fm \quad \text{(i=1, 2)} \tag{8}$$

The universal function $\Phi(s)$ reads as:

$$\Phi(s) = \left[A \exp\left(\frac{s}{d_1}\right) + B \exp\left(\frac{s}{d_2}\right) \right]^{-1} \tag{9}$$

with A = 0.0300 MeV^{-1}fm, B = 0.0061 MeV^{-1}fm, d_1 = 3.30fm and d_2 = 0.65 fm.

1.4 Bass 1980

The above potential was further improved by Bass [14] and is given as,

$$\Phi(s) = \left[0.033 \exp\left(\frac{s}{3.5}\right) + 0.007 \exp\left(\frac{s}{0.65}\right) \right]^{-1} \tag{10}$$

Here R_i is taken as:

$$R_i = R_s \left(1 - \frac{0.98}{R_s^2}\right) \qquad (i=1, 2) \tag{11}$$

1.2 Broglia and Winther 1991 (BW 91)

In BW 91 [14], the nuclear potential is given as,

$$V_N(r) = -\frac{V_0}{1 + \exp\left(\dfrac{r - R_0}{0.63}\right)} MeV \tag{12}$$

with $V_0 = 16\pi \dfrac{R_1 R_2}{R_1 + R_2} \gamma a$ (13)

Here a = 0.63 and $R_0 = R_1 + R_2 + 0.29$

The radius, R^i, has the form:

$$R_i = 1.233\, A_i^{1/3} - 0.98\, A_i^{-1/3} fm\, (i = 1,2) \tag{14}$$

The form of the surface energy coefficient γ is taken as:

$$\gamma = \gamma_0 \left[1 - k_s \left(\frac{N_d - Z_d}{A_d}\right)\left(\frac{N_c - Z_c}{A_c}\right) \right] \tag{15}$$

Where γ_0 = 0.95 MeV/fm^2 and k_s =1.8.

III. Results and Discussions

The heavy particle decay studies from the isotopes of $^{216\text{-}226}$Fr have been performed using CPPM, Bass 1973, Bass 1977, Bass 1980 and BW 1991. In order to have a heavy particle decay process, the energy of the reaction, Q value, must be greater than zero (Q > 0). Since the heavy particle decay is not accompanied by the emission of any neutrons, the Q value of the reaction goes completely into the total kinetic energy of the decay products; the daughter nucleus and the emitted cluster.

The energy released in the decay process is given as,

$$Q = \Delta M_p - (\Delta M_c + \Delta M_d) + k(Z_p^{\varepsilon} - Z_d^{\varepsilon}) \tag{16}$$

where ΔM_p, ΔM_d, ΔM_c are the mass excess of the parent, daughter and emitted cluster respectively. The term $k(Z_p^{\varepsilon} - Z_d^{\varepsilon})$ represents the screening effect of atomic electrons [17], where k = 8.7eV , ε = 2.517 for $Z \geq 60$ and k = 13.6eV, ε = 2.408 for $Z <$ 60. These values have been derived from data reported by Huang et al., [18].

We have evaluated the decay half lives for the emission of ^{14}C cluster from $^{216\text{-}226}$Fr within CPPM and also within the nuclear potentials Bass 1973, Bass 1977, Bass 1980 and BW 91. For a comparative study, the evaluated values using different nuclear potentials are listed in TABLE 1. The study has shown that the values obtained using CPPM and Bass 1973 are in excellent with the experimental data available. All other potentials, Bass 1977, Bass 1980 and BW 1991 also matches with the experimentally observed value, but with a one order difference. Consider, the emission of ^{14}C cluster from ^{221}Fr. Using CPPM, the half life for the above heavy particle decay is $2.960x10^{14}$s and the values obtained using Bass 1973, Bass 1977, Bass 1980 and BW 91 are $3.057x10^{14}$s, $9.648x10^{13}$s, $1.344x10^{13}$s and $7.605x10^{15}$s respectively. When compared with the experimental value, $3.311x10^{14}$s, the values obtained using CPPM and Bass 1973 are found to be in excellent agreement with the experimental value, whereas in all other cases we have noticed a one order difference in half lives. This shows the predictive power and reliability of our model CPPM in the heavy particle decay studies.

Table 1. The comparison of estimated half lives for the emission of ^{14}C cluster from $^{216\text{-}226}$Fr using different nuclear potentials.

Parent nuclei	Cluster	Q value (MeV)	$T_{1/2}$(s)					Expt.
			CPPM	Bass 1973	Bass 1977	Bass 1980	BW 91	
^{216}Fr	^{14}C	26.046	$4.498x10^{28}$	$4.667x10^{28}$	$3.246x10^{28}$	$3.512x10^{27}$	$2.872x10^{29}$	
^{217}Fr	^{14}C	27.165	$2.400x10^{25}$	$2.549x10^{25}$	$1.535x10^{25}$	$1.729x10^{24}$	$1.996x10^{26}$	
^{218}Fr	^{14}C	28.494	$5.404x10^{21}$	$5.831x10^{21}$	$2.971x10^{21}$	$3.521x10^{20}$	$6.383x10^{22}$	
^{219}Fr	^{14}C	29.529	$1.096x10^{19}$	$1.180x10^{19}$	$5.196x10^{18}$	$6.442x10^{17}$	$1.706x10^{20}$	
^{220}Fr	^{14}C	30.825	$7.174x10^{15}$	$7.549x10^{15}$	$2.714x10^{15}$	$3.619x10^{14}$	$1.577x10^{17}$	
^{221}Fr	^{14}C	31.401	$2.960x10^{14}$	$3.057x10^{14}$	$9.648x10^{13}$	$1.344x10^{13}$	$7.605x10^{15}$	$3.311x10^{14}$
^{222}Fr	^{14}C	30.188	$1.873x10^{17}$	$1.989x10^{17}$	$7.353x10^{16}$	$9.390x10^{15}$	$3.505x10^{18}$	
^{223}Fr	^{14}C	29.111	$7.817x10^{19}$	$8.407x10^{19}$	$3.476x10^{19}$	$4.170x10^{18}$	$1.104x10^{21}$	
^{224}Fr	^{14}C	28.130	$2.508x10^{22}$	$2.700x10^{22}$	$1.222x10^{22}$	$1.393x10^{21}$	$2.743x10^{23}$	
^{225}Fr	^{14}C	26.990	$3.030x10^{25}$	$3.231x10^{25}$	$1.623x10^{25}$	$1.752x10^{24}$	$2.466x10^{26}$	
^{226}Fr	^{14}C	26.180	$5.935x10^{27}$	$6.256x10^{27}$	$3.354x10^{27}$	$3.485x10^{26}$	$3.952x10^{28}$	

The entire calculations and comparisons are displayed in Fig.1, the plot of $\log_{10}T_{1/2}$ against the neutron number of the daughter nuclei for the emission of the cluster ^{14}C from $^{216\text{-}226}$Fr isotopes. It is to be noticed from Fig. 1 that all the nuclear potentials under study and CPPM follow the same trend. In addition, we have observed a prominent dip at N=126. A minimum in the decay half lives corresponds to the greater barrier penetrability, which in turn indicates the neutron shell closure of the daughter nuclei.

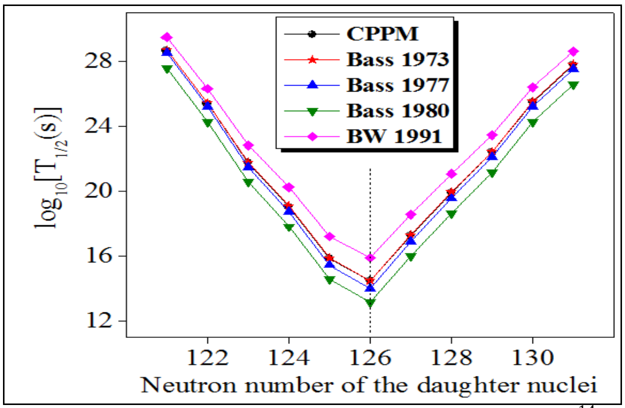

Fig.1. Plot of $\log_{10}T_{1/2}$(s) vs neutron number of daughter nuclei for ^{14}C from $^{216\text{-}226}$Fr.

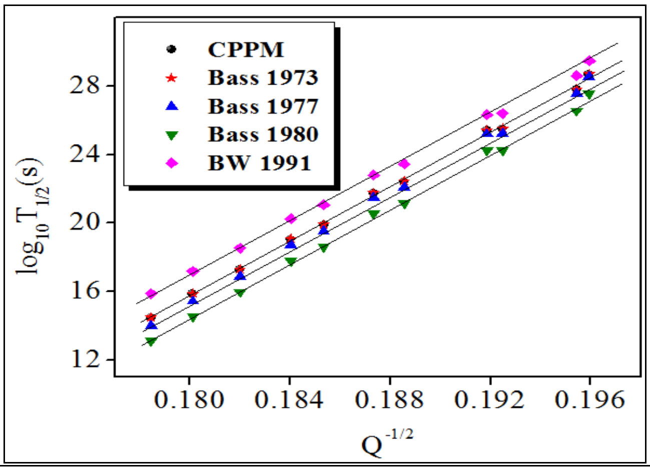

Fig.2. Geiger-Nuttall plot for the emission of ^{14}C from $^{216\text{-}226}$Fr.

The G-N plots, for the ^{14}C emission from $^{216\text{-}226}$Fr, have been drawn for all models and are displayed in Fig.2 and Fig.3. The G-N plots are found to be linear with same slope and different intercepts for CPPM and other nuclear potentials. From the nature of the plots we can say that the inclusion of proximity potential will not produce much deviation to the linearity of the GN plots. In Fig.4 we have shown the plot between the negative logarithm of penetrability (−ln P) and the logarithmic half-lives ($\log_{10}T_{1/2}$), Universal curve, for the emission of ^{14}C cluster. The obtained graph is linear for all nuclear potentials with almost same slope (slope = 0.434 and intercept = -20.159).

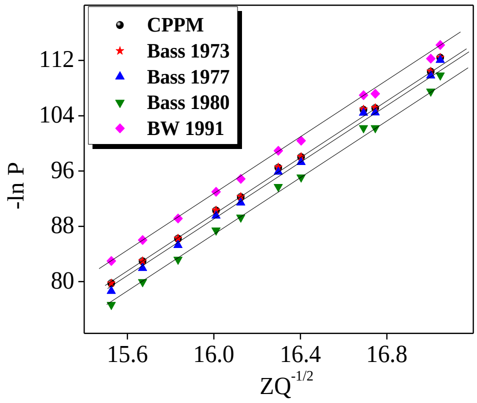

Fig.3. G-N plot for the emission of ^{14}C from $^{216\text{-}226}$Fr.

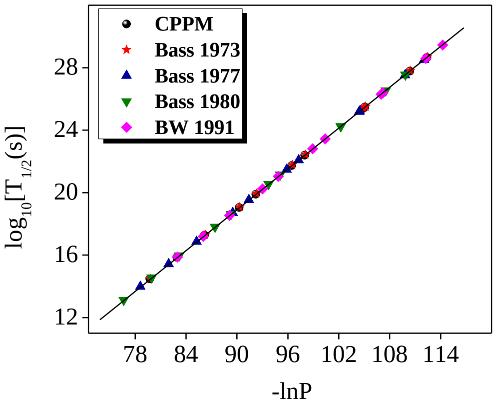

Fig.4. Universal curve of $\log_{10}T_{1/2}(s)$ versus -lnP for ^{14}C from $^{216\text{-}226}$Fr.

IV. Conclusions

The heavy particle decay of $^{216\text{-}226}$Fr has been studied using CPPM, Bass1973, Bass 1977, Bass 1980 and BW 91 by calculating the decay half lives. The estimated half lives are observed to show the same behavior in CPPM and other nuclear potentials. On comparison with the available experimental data, CPPM and Bass 1973 showed an excellent agreement whereas there observed a one order difference in half lives in the case of other nuclear potentials. We could also point out the existence of the neutron magic number at N=126 from the plot of $\log_{10}T_{1/2}$ versus neutron number of the daughter nuclei. The G-N plot is obtained with same slope and different intercepts for different nuclear potentials and the Universal curve is obtained as linear for all nuclear potentials. The linearity of G-N plot and Universal curve and the experimental matching of half lives well establish the strength of CPPM in heavy particle decay studies.

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